Exponential Diversity Achieving Spatio-Temporal Power Allocation Scheme for MIMO Fading Channels

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Abstract — **We analyze optimum (space–time) adaptive power transmission policies for Rayleigh fading MIMO channels when CSIT and CSIR are available. We show that our power allocation policy provides exponential diversity gain[2] (BER $\leq \alpha e^{-f(n_t,n_r)}$, where $\alpha > 0$ is a constant, and $f > 0$ is an increasing function of n_t & n_r) if perfect CSIT is available. Exponential diversity is lost at high SNR if the quality of CSIT degrades.**

I. Perfect/Imperfect CSIT

We consider a single user narrowband (*flat fading*) communication system employing n_t transmit antennas and n_r receive antennas. The channel between i^{th} receive antenna and j^{th} transmit antenna, h_{ij} is a complex Gaussian random variable ($H = [h_{ij}]$ represents the channel). We assume i.i.d. Rayleigh fading from symbol to symbol and on each of the diversity branches. The additive noise, n, is temporally and spatially white with mean zero, i.e., $n \sim \mathcal{N}_C(0, \sigma^2 \mathrm{I}_{n_r})$. We assume that $\hat{H}$ is the transmitter's estimate of the channel. We assume that $\hat{H}$ and H are jointly complex Gaussian with correlation ρ. We assume perfect CSIR. $\hat{H}$ is used to get the optimal beamforming transmit weight vector w (the eigenvector of $\hat{H}^H\hat{H}$ corresponding to its largest eigenvalue) and transmit power $P(.)$ for that symbol duration. The output of the matched filter sampled at symbol duration is given by $y = \sqrt{P(\hat{\gamma})}\; H\; w\; x + n$, where x is the transmitted symbol, $\gamma = \|Hw\|^2 \mathrm{E}|x|^2/\sigma^2$ is the SNR, $P(\hat{\gamma})$ is the transmit power, and $\hat{\gamma}$ $(= \|\hat{H}w\|^2\mathrm{E}|x|^2/\sigma^2)$ is the estimate of γ at the transmitter.

The BER performance of the above system for the coherent BPSK signaling is given by $P_{e|\gamma,\hat{\gamma}} = Q\left(\sqrt{2\gamma\; P(\hat{\gamma})}\right)$. We minimize P_e subject to the average transmit power constraint. For the perfect CSIT case ($\hat{\gamma} = \gamma$), the optimization problem is $\min_{E_\Gamma[P(\gamma)]\leq 1} \int_0^\infty \mathrm{Q}\left(\sqrt{2\gamma P(\gamma)}\right) f_\Gamma(\gamma) d\gamma$, where f_Γ is the pdf of γ. Replacing Q(x) by a tight upper bound $\frac{1}{2}e^{-x^2/2}$ and applying Lagarange's method, we obtain the optimum solution

$$P(\gamma) \;=\; \begin{cases} \frac{1}{\gamma}\ln\left(\frac{\gamma}{\gamma_0}\right) & \text{for } \gamma \geq \gamma_0 \\ 0 & \text{for } \gamma < \gamma_0, \end{cases} \qquad (1)$$

where γ_0 is the solution of $\int_{\gamma_0}^\infty P(\gamma) f_\Gamma(\gamma) d\gamma = 1$. For the imperfect CSIT case, we replace γ by $\hat{\gamma}$ in Eqn. 1. It should be noted here that the transmitter should have the knowledge of the fading statistics to compute γ_0 and estimate of instantaneous fade values $\hat{H}$ to compute $P(\hat{\gamma})$.

We evaluated the upper bounds of the BER for the perfect and imperfect CSIT cases and the results are tabulated below.

Perfect CSIT

$$\text{SISO} \quad P_e \;\leq\; k\,[1+k]\,e^{-\sqrt{2s}}$$

$$\text{MISO} \quad P_e \;\leq\; 0.5\tfrac{n_t}{n_t-1} k e^{-(n_t-1)s+\delta}$$

$$\text{MIMO} \quad P_e \;\approx\; 0.5 K e^{-s/K} \qquad (2)$$

Imperfect CSIT

$$\text{SISO} \quad P_e \;\leq\; k e^{-\sqrt{2s}} + k e^{-\sqrt{2s}} \frac{e^{-\alpha'}}{\alpha'}$$

$$\text{MISO} \quad P_e \;\leq\; 0.5 k e^{-(n_t-1)s+\delta}\left[1 + \frac{e^{-(n_t-1)^2 s + (n_t-1)\delta}}{\alpha'^{\,n_t}}\right]$$

where s $= \mathrm{E}|x|^2/\sigma^2$ is the average SNR per branch, $\delta = \sum_{m=0}^{n_t-3}\frac{1}{m+1}$, $\alpha' = \left[1 + \frac{s}{\rho^2}\right]\frac{\gamma_0}{s} + \frac{\left(\frac{\gamma_0}{s}\right)^2 \frac{s}{\rho^2}}{\frac{\gamma_0}{s} + \frac{1}{es}}$, and k is a constant satisfying -ln(1-x) $\leq kx$. The value of K depends on n_t and n_r and is explicitly computed in the detailed version of the paper. From Eqn. 2, we see that we obtain exponential diversity for perfect CSIT. For imperfect CSIT, the second term dominates at high SNR causing a loss of exponential diversity.

II. Simulations and Extensions

Fig. 1 shows the BER performance of *beamforming* with *space-time power allocation (STPA)* for an uncoded system having $n_t = 4$ and $n_r = 2$. Fig. 2 shows the BER performance of a coded system with an outer rate-1/2 *convolutional code* and an inner *Alamouti* code for $n_t = 2$ and $n_r = 1$. We observe that for the perfect CSIT case, our power allocation policy provides *exponential order diversity* gain for both the coded and uncoded systems which is substantially more than the conventional *space-only (SOPA)* [1] and *uniform* power allocation schemes. Also, when the quality of CSIT degrades ($\rho \neq 1$), the exponential diversity is lost at high SNR for our policy. The interesting observation here is that we still achieve exponential diversity at low SNR (up to 9 dB in the systems we studied). In Fig. 2, we note that at $\rho = 1$ and $P_e = 10^{-3}$ the gain in STPA compared to SOPA is 2.5 dB and at $\rho = 0.99$ the gain is 3.2 dB. The channel estimation in GSM is done by a 26-bit midamble sequence. A ML estimate of H from this sequence can provide ρ as large as 0.99 for a SNR of 10 dB. Furthermore, since the fading is i.i.d., interleaving will not improve the performance.

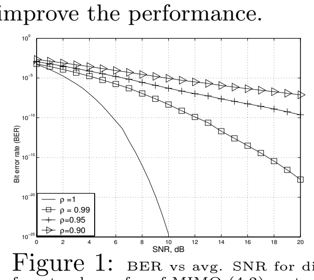

Figure 1: BER vs avg. SNR for different values of ρ of MIMO (4,2) system.

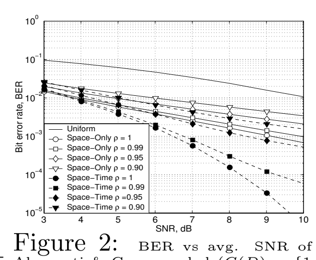

Figure 2: BER vs avg. SNR of a Alamouti & Conv. coded ($G(D) = \{1 + D^2, 1 + D + D^2\}$) (2,1) system

References

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[2] A. Rangarajan, and V. Sharma, "Achieving Exponential Diversity Order in Rayleigh Fading Channels," *National Conf. on Commun.,* Bangalore, 2004.

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[2]The BER decays as $\mathrm{SNR}^{-n_t n_r d_{min}}$ for conventional *beamforming* systems with an outer fully interleaved convolutional code whose *minimum distance* is d_{min} ($d_{min} = 1$ for an uncoded system).