Energy Optimal Packet Scheduling with Individual Packet Delay Constraints

Dony J. Muttath, M. Santhoshkumar, and K. Premkumar
Department of Electronics and Communication Engineering
Indian Institute of Information Technology
Design and Manufacturing, Kancheepuram
Chennai 600 127, India.
E–mail: kpk@iiitdm.ac.in

***Abstract*—We consider a problem of scheduling packets in a fading link where each packet is required to reach the destination before a delay deadline. Time is slotted, and at the beginning of each slot, a packet arrives according to an arrival process. A packet on arrival is stored in a buffer, and it is required to reach the destination before a total delay of $d + 1$ slots (i.e., the maximum waiting time in the buffer can be d time-slots and one time–slot delay for transmission). At the beginning, the transmitter is provided with a finite energy E_0, and the problem that we consider is to obtain an optimum scheduler that decides which time–slots to be used for transmission such that it maximizes the number of packet transmissions with a total energy E_0, and within a delay deadline of $d+1$ time-slots for each packet. We model this problem as a Markov Decision process, and provide a dynamic programming (DP) based solution which is prohibitively complex, but can be numerically solved. The computational complexity of the DP solution motivates us to provide two sub-optimal heuristic solutions to the scheduling problem. We provide the throughput and average energy performance of the heuristic solutions.**

***Index Terms*—delay deadline, scheduling, throughput optimal scheduling, scheduling with finite energy**

I. Introduction

Wireless systems are often hand–held systems, and thus, are almost always constrained by energy. Over the last few years, a lot of work has been done in improving the energy efficiency of wireless systems. All the works done so far consider energy–efficiency from an information theoretic point of view where the rate can be adapted based on the channel conditions. However, we deviate from the existing work in the following sense: we do not adapt the rate, but select the appropriate time–slots for packet transmission. In each time–slot chosen for transmission, we transmit a packet with a power that is just enough to achieve a required packet error rate.

We consider a delay sensitive communication in this paper. In general, each packet is associated with a time–to–live (TTL) parameter, and thus, it is important to schedule transmission of a packet before it's delay deadline; otherwise, the packet is lost. Thus, we have conflicting objectives: maximizing the number of packet transmissions within a delay deadline, and minimizing the average energy of transmission. In this work, we pose the scheduling problem described above, and obtain an optimum scheduling procedure for packet transmissions.

Previous work: A plethora of work has been done in this area so far, and we highlight important contributions that achieve energy efficiency.

Transmission policies that minimize the average power spent in each slot is studied in [1]. Using a dynamic programming formulation, it is shown that the optimum policy transmits a packet if the backlog is larger than a critical value, and that the critical backlog value depends on the channel state. It is to be noted here that *this problem considers an average delay constraint rather than an individual constraint on each packet*.

In [2], a single link with a perfect channel state information (CSI) is considered. The link undergoes fading. A buffer at the transmitter stores data, and the transmission rate and power are adapted according to the CSI. The objectives in [2] are *minimizing both the average delay and the average power, and hence, the authors minimize a weighted combination of the average delay and average power*. A dynamic programming formulation is obtained that provides all Pareto optimal (power, delay) points. An optimal power–delay tradeoff is obtained that shows the power requirement can be made smaller as more delay is tolerated.

In [3], the authors consider a problem of scheduling M packets within a total time duration of T. An optimum offline scheduler is obtained, and an online scheduler (based on the optimum offline scheduler) is proposed. It is shown through simulations that the proposed online scheduler performs very close to the optimum offline scheduler.

In [4], Fu et al., considers the a throughput maximisation problem over n time–slots with a constraint on energy. This work, however, has a fluid rate for each time slot, which is obtained using Shannon's capacity formula.

[5] generalizes the problem studied in [3] with a delay deadline constraint on each packet. An optimal offline scheduler is proposed, and the stability and packet delay analysis of the optimal scheduler are studied.

In [6], Chen et al., considers the problem of transmitting M packets through an additive white Gaussian noise channel, with individual packet delay constraints. However, the problem considered is an optimum offline scheduling with continuous time. Again, an information theoretic power–rate function is used to arrive at an optimum offline scheduler.

978-1-5386-8134-3/18/$31.00 ©2018 IEEE

In [7], the problem of energy–efficient scheduling with individual packet delay constraints is studied. The authors pose this as a convex optimization problem and obtain an optimal offline scheduler, and have shown the packet delay properties of the optimal scheduler.

In [8], an optimum rate control problem is studied in which B units of data has to be transmitted by deadline T using as low energy as possible. Again, the problem uses Shannon's capacity formula to arrive at an optimum solution.

In [9], the problem of a single transmitter communicating to n different receivers, with delay constraints is studied. In particular, the problem is posed in a convex optimisation framework, and a rate optimal strategy is derived. The objective function is the energy spent for transmission along with the energy required to drive the associated RF circuit. It has been shown that the power required to drive the RF circuit puts a limit on the minimum energy efficient rate.

[10] considers the following scheduling problem: a packet of B bits has to be transported within T time slots through a fading channel in an energy efficient manner. They formulated the problem as a Markov Decision Process, and obtained an optimal scheduling policy only for a special case of $T = 2$. Also, suboptimal policies are derived for bit allocation.

Lowering the transmission rate, and scheduling transmission based on channel gain improves the energy efficiency of a wireless transmitter, but with large delays. Based on channel gain, one can vary transmission rate and power in a wireless network, and can utilize the resources more efficiently. However, such an approach may lead to large delays, or buffer overflows [11].

Contributions of the paper: In all the works that we have discussed, the energy (or power) model is taken from Shannon's capacity formula for additive white Gaussian noise (AWGN) channels with fading. Then, energy is shown as a convex increasing function of rate, and the solution to energy minimization problem is the same as having a maximum transmission time for each transmission opportunity. However, in a practical communication system, time is slotted, and hence, allowing an arbitrary time for a transmission opportunity may not always be a good proposal.

It is to be noted that Shannon's capacity formula is based on a random coding argument which is valid when the codeword length goes to infinity. For a finite length codeword (or for a finite channel use), this would not make much sense. For this reason, we restrict ourselves to a simple signal–to–noise ratio (SNR) model, which prescribes a certain SNR to achieve a certain target packet error rate.

In this paper, we propose the optimum scheduler based on dynamic programming. Since, the optimum scheduler has a prohibitively high computational complexity, we propose two sub–optimal schedulers: the first procedure is based on a threshold policy on channel gain, and the second procedure is based on probabilistically using the channel, where the probability of using the channel increases with the channel gain.

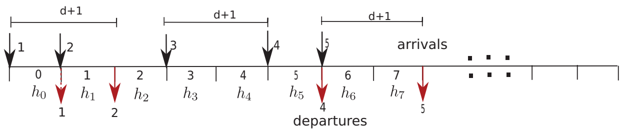

Fig. 1. An illustration of the scheduling procedure for a delay deadline of $d = 1$ slot (i.e., a packet can remain in the system for at most 2 slots). Packets 1, 2, 3, 4, and 5 arrive at the beginning of time–slots 0, 1, 3, 5, and 6, respectively. Note that the packets 1, 2, 4, and 5 depart at the end of time–slots 0, 1, 5, and 7, respectively. Packet 3 (that has arrived at the beginning of time–slot 3) is dropped, as the channel gain during slots 4 and 5 are very bad for transmission.

Organization of the paper: In Section II, we define the system model. In Section III, we formulate a scheduling problem that seeks to maximize throughput of packets, when each of the packets are constrained to reach the destination within a delay bound. In Section IV, we propose heuristic solutions to the problem. In Section V, we provide numerical results, and discuss the results. In Section VI, we provide conclusions, and scope for future work.

II. System Model

We consider a single transmitter which sends packets to a receiver. To begin with, the transmitter is equipped with an energy of E_0, and it is desirable to use this energy to transmit a maximum number of packets. However, each packet is required to be received within a certain delay. It is to be noted that the link between the transmitter and the receiver undergoes fading which makes the problem more interesting.

We consider a time–slotted system in which each time–slot is of unit length. The length of a time–slot is the same as the length of a packet[1] (measured in units of time for transmission). Thus, a time–slot k takes non–negative integer values.

We consider an i.i.d. block fading channel, in which the fading gain process $\{H[k] : k \in Z_+\}$ varies independently across slots indexed by k, and in each time–slot, it remains the same. It is to be noted that when $H[k]$ is very small, the transmitter needs to transmit with a large power for successful reception of the packet.

At the beginning of each time–slot k, a packet arrives according to a Bernoulli distribution with the probability of a packet arrival being $p \in (0, 1)$. Let the i.i.d. Bernoulli random variables $A[k]$ denote whether a packet has arrived or not at the beginning of time–slot k. A packet on arrival is immediately stored in a transmit–buffer. We note that when $H[k]$ is very close to zero, it is not optimal for the transmitter to transmit the packet immediately, as it requires a large transmit energy to achieve a target SNR, and thus, reduces the number of packets that can be transmitted. Thus, during each time–slot k, the transmitter makes the decision $D[k]$ of *transmitting a packet (denoted by '1')* or *not–transmitting a packet (denoted by '0')*. Note that when the transmit–buffer is empty, the decision $D[k]$ is always 0.

[1]All packets are assumed to have the same length.

Thus, the evolution of the queue–length in the transmit–buffer is given by the following. At the beginning of time–slot k, let $Q[k]$ be the number of packets waiting for transmission in the transmit–buffer. For the case of packets with no delay constraints, the queue–length evolution is given by,

$$Q[k+1] = Q[k] - D[k] + A[k+1].$$

However, each packet is required to be received before a deadline $d+1$. If a packet is not received before $d+1$ time–slots from the arrival epoch, it will be dropped. Let $W_i[k], i = 1, 2, \cdots, Q[k]$ be the waiting time of packet i (where packets are numbered starting from the head–of–line packet) in the queue at the beginning of time–slot k, and define $\boldsymbol{W}[k] = [W_1[k], W_2[k], \cdots, W_{Q[k]}[k]]$. Note that when a packet has just arrived, it's waiting time is zero. Thus, the queue is described by a tuple $(Q[k], \boldsymbol{W}[k])$.

Deadline and Transmission Opportunity: We consider a deadline constraint in which the delay deadline parameter $d \in \{0, 1, 2, \cdots\}$ is the same for all packets. Thus, if there is an arrival at time–slot u, the transmitter can choose a slot from $\{u, u+1, u+2, \cdots, u+d\}$ for transmission of the packet. If for some reason, the packet is not transmitted in any of the slots in $\{u, u+1, u+2, \cdots, u+d\}$, the packet is discarded, which is considered as a packet loss.

If $W_1[k] = d$ and $D[k] = 0$, the head–of–line (HOL) packet has been in transmit–buffer for $d+1$ time–slots at the beginning of time–slot $k+1$, and hence, will be dropped at time–slot $k+1$. Thus,

$$Q[k+1] = \begin{cases} Q[k] - D[k] + A[k+1], & \text{if } W_1[k] < d, \\ Q[k] - 1 + A[k+1], & \text{if } W_1[k] = d. \end{cases} \tag{1}$$

The waiting time evolution is given by the following. If there is no new arrival, i.e., $A[k+1] = 0$, then $\forall i = 1, 2, \cdots, Q[k+1]$,

$$W_i[k+1] = \begin{cases} W_{i+1}[k] + 1, & \text{if } W_1[k] = d \text{ or } D[k] = 1, \\ W_i[k] + 1, & \text{otherwise.} \end{cases} \tag{2}$$

If there is a new arrival, i.e., $A[k+1] = 1$, then the waiting time of the packet that has just arrived, $W_{Q[k+1]}[k+1] = 0$, and for packets $i = 1, 2, \cdots, Q[k+1] - 1$,

$$W_i[k+1] = \begin{cases} W_{i+1}[k] + 1, & \text{if } W_1[k] = d \text{ or } D[k] = 1, \\ W_i[k] + 1, & \text{otherwise.} \end{cases} \tag{3}$$

Let $E[k]$ be the energy that the transmitter has at the beginning of time–slot k. Thus, at time–slot $k = 0$, $E[0] = E_0$, the initial energy of the transmitter. During each time–slot k, the transmitter can choose to transmit a packet (if $Q[k] > 0$ and the available energy $E[k]$ is good enough to transmit a packet), in which case, the transmitter spends some energy $\Lambda[k] > 0$; otherwise, $\Lambda[k] = 0$. Also, we assume that when $E[k_1] < \epsilon$, then there is no subsequent transmission at all, i.e., for all $k \geq k_1$, $D[k] = 0$.

We assume that a packet transmission is successful only if the SNR is at least as large as γ (a threshold that depends on the target packet error rate). For any modulation and coding scheme, it is found that the packet error probability decreases monotonically with SNR, and hence, for the target packet error rate, it is enough to transmit with an energy that achieves a minimum required SNR for the target packet error rate.

The transmit power during time–slot k is given by

$$\lambda[k] = \begin{cases} \frac{\sigma^2 \gamma}{H[k]^2}, & \text{if } D[k] = 1, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

where σ^2 is the variance of the additive white Gaussian noise of the channel. Thus, the energy available at the beginning of time–slot $k+1$ is given by

$$E[k+1] = E[k] - \lambda[k]. \tag{5}$$

We note that (4) holds good for one channel use (i.e., for one symbol). However, we consider a block fading channel in which the channel gain $H[k]$ is the same for all bits of a packet (which fits one slot), (4) gives the power required for all the bits of a packet. Since, the length of a time–slot is normalized to unity, this gives the energy required to transmit one packet.

III. Energy Optimal Delay Constrained Scheduling Problem

The problem that we consider is one of maximizing the number of packets transmitted, subject to an individual packet delay constraint, and with a constraint on total transmit energy.

Recall from Section II that at the beginning of slot k, $Q[k]$ is the number of packets waiting in the transmit–buffer for transmission, and each packet i (starting from HOL packet) has been waiting in the queue for $W_i[k]$ time–slots. Note that $W_1[k]$ is the waiting time of the earliest packet (which is the HOL packet) that is still waiting for service, and $W_{Q[k]}[k]$ is the waiting time of the packet that has most recently joined the queue.

We define the state of the system at the beginning of time–slot k, as follows:

$$\boldsymbol{X}[k] = [E[k], Q[k], \boldsymbol{W}[k], H[k]]. \tag{6}$$

Also, at the beginning of each slot k, the transmitter has to make one of the following decisions in $\{0, 1\}$, where the decision $D[k] = 0$ means that there is no transmission attempt, and $D[k] = 1$ means that there is a transmission attempt.

Based on the current state and the current decision (or action) taken, the system goes to a next state, showing that it is a discrete–time dynamical system. Thus, one can associate rewards for choosing an action in a state, and formulate the maximum throughput problem as a Markov Decision Process.

The one stage reward function for the problem is given by the following. Let $\boldsymbol{x} = [e, q, \boldsymbol{w}, h]$ be a state, and a be an action. The reward is given by

$$r(\boldsymbol{x}, a) = \begin{cases} 1, & \text{if } e \geq \frac{\sigma^2 \gamma}{h^2}, q > 0, \text{and } a = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

The maximization problem is

$$\max_{\{D[0], D[1], \cdots\}} \mathbb{E}\left[\sum_{k=0}^{\infty} r(\boldsymbol{X}[k], D[k]) \middle| \boldsymbol{X}[0]\right] \tag{8}$$

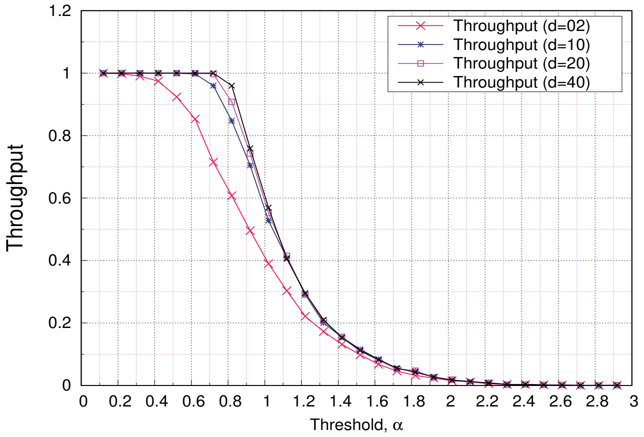

Fig. 2. Throughput of Threshold based Scheduler vs Threshold α

which can be solved using Bellman's equation,

$$J(\boldsymbol{x}) = \max_{a \in \{0,1\}} r(\boldsymbol{x}, a) + \mathbb{E}\left[J\left(\boldsymbol{x}'\right) \middle| \boldsymbol{x}, a\right], \qquad (9)$$

where $\boldsymbol{x}' = [e', q', \boldsymbol{w}', h']$ is the next state of the system, given that the current state is $\boldsymbol{x} = [e, q, \boldsymbol{w}, h]$, and the current action is a. We note that the state evolution is described in Section II.

IV. Sub-optimal Schedulers

Solving (9) is an extremely complex problem (even by numerical methods like value iteration). Hence, we provide sub–optimal methods based on heuristics for online scheduling.

A. Threshold Based Scheduler

We propose an online scheduler that is based on the channel gain during each time–slot.

This algorithm is motivated by the following: when the channel gain is above a certain threshold α, the link is used for packet transmission, i.e.,

$$D[k] = \begin{cases} 1, & \text{if } H[k] \geqslant \alpha, Q[k] > 0, \\ 0, & \text{otherwise.} \end{cases} \qquad (10)$$

The transmit power that achieves an SNR of γ is chosen for transmission. If the threshold α is chosen to be small, then a large energy is spent when the channel gain is close to α, and thus, one may not achieve a high throughput. But, this has the advantage of achieving a small delay and a small outage. On the other hand, if α is large, then less energy is spent for packet transmissions. However, this may overshoot the delay requirement, and hence, may perform poor in terms of outage. The algorithm based on threshold on channel gain is described in **Algorithm 1**.

B. Probabilistic Scheduler

This algorithm is motivated by the following: when the channel gain $H[k]$ is large, the link can be used with a large probability, and vice–versa. Thus, we are interested in a probability mass function for a given channel gain H[k] which monotonically increases with $H[k]$. We choose the following function

$$\mathbb{P}\{\text{Transmission} \mid \text{Channel gain is } H[k]\} = 1 - \exp\left(-H[k]^2\right).$$

The algorithm is described in **Algorithm 2**.

Algorithm 1 Threshold Based Scheduler

Input: d, E_0, γ, σ, $\{A[k] : k \in \mathbb{Z}_+\}$, $\{H[k] : k \in \mathbb{Z}_+\}$, ϵ, α
Output: η, o
Initialisation :
Throughput $\eta \leftarrow 0$
Packet loss $o \leftarrow 0$
$k \leftarrow 0$
$E[k] \leftarrow E_0$
$Q[k] \leftarrow 0$
$W[k] \leftarrow []$
LOOP Process
while $E[k] \geq \epsilon$ **do**

  if $(A[k] = 1)$ **then**
    $Q[k] \leftarrow Q[k] + 1$
    $W_{Q[k]}[k] \leftarrow 0$
  end if

  if $(H[k] \geqslant \alpha)$ and $\left(E[k] \geqslant \frac{\sigma^2\gamma}{H[k]^2}\right)$ **then**
    Transmit HOL packet from transmit–buffer
    $\eta \leftarrow \eta + 1$
    $E[k+1] = E[k] - \frac{\sigma^2\gamma}{H[k]^2}$
    for $i = 1$ to $Q[k] - 1$ **do**
      $W_i[k+1] \leftarrow W_{i+1}[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k] - 1$
  else if $W_1[k] = d$ **then**
    Drop HOL packet from transmit–buffer
    $o \leftarrow o + 1$
    $E[k+1] = E[k]$
    for $i = 1$ to $Q[k] - 1$ **do**
      $W_i[k+1] \leftarrow W_{i+1}[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k] - 1$
  else
    $E[k+1] = E[k]$
    for $i = 1$ to $Q[k]$ **do**
      $W_i[k+1] \leftarrow W_i[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k]$
  end if
  $k \leftarrow k + 1$
end while
return $\frac{\eta}{\eta + o}$

V. Numerical Results

In this Section, we evaluate the throughput and average energy used for transmission for scheduling packets in a fading

Algorithm 2 A Probabilistic Scheduler

Input: d, E_0, γ, σ, $\{A[k] : k \in \mathbb{Z}_+\}$, $\{H[k] : k \in \mathbb{Z}_+\}$, ϵ
Output: η, o

Initialisation :
Throughput $\eta \leftarrow 0$
Packet loss $o \leftarrow 0$
$k \leftarrow 0$
$E[k] \leftarrow E_0$
$Q[k] \leftarrow 0$
$W[k] \leftarrow []$
LOOP Process
while $E[k] \geq \epsilon$ **do**

  if $(A[k] = 1)$ **then**
    $Q[k] \leftarrow Q[k] + 1$
    $W_{Q[k]}[k] \leftarrow 0$
  end if
  $G[k] \leftarrow H[k]^2$
  $p = 1 - \exp(-G[k])$
  Generate a Bernoulli random variable D with $\mathbb{P}\{D = 1\} = p$

  if $D = 1$ and $\left(E[k] \geqslant \frac{\sigma^2\gamma}{H[k]^2}\right)$ **then**
    Transmit HOL packet from transmit–buffer
    $\eta \leftarrow \eta + 1$
    $E[k+1] = E[k] - \frac{\sigma^2\gamma}{H[k]^2}$
    for $i = 1$ to $Q[k] - 1$ **do**
      $W_i[k+1] \leftarrow W_{i+1}[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k] - 1$
  else if $W_1[k] = d$ **then**
    Drop HOL packet from transmit–buffer
    $o \leftarrow o + 1$
    $E[k+1] = E[k]$
    for $i = 1$ to $Q[k] - 1$ **do**
      $W_i[k+1] \leftarrow W_{i+1}[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k] - 1$
  else
    $E[k+1] = E[k]$
    for $i = 1$ to $Q[k]$ **do**
      $W_i[k+1] \leftarrow W_i[k] + 1$
    end for
    $Q[k+1] \leftarrow Q[k]$
  end if
  $k \leftarrow k + 1$
end while
return $\frac{\eta}{\eta + o}$

link. The delay constraint d varies from 2 to 40, and the target SNR γ is taken as 24dB. The channel noise variance σ^2 is taken as 0.3 and the probability of new arrival $\lambda = 0.5$.

Throughput is computed as the long range fraction of the number of packets transmitted to the number of arrivals, and average transmit energy is computed for the number of transmitted packets. Throughput and average energy are plotted as a function of threshold α, as shown in Figure 2 and Figure 3. As the delay constraint d increases from 2 to 40, throughput also increases for a given threshold (see Figure 2). This is because, for a large d, the probability of finding a slot with a channel gain larger than α is large, and thus for $d = 40$, a packet is more likely to see a better channel gain than for $d = 2$.

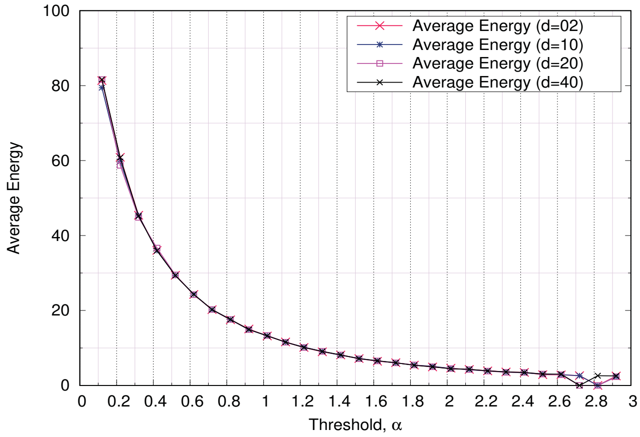

Fig. 3. Average Energy of Threshold based Scheduler vs Threshold α

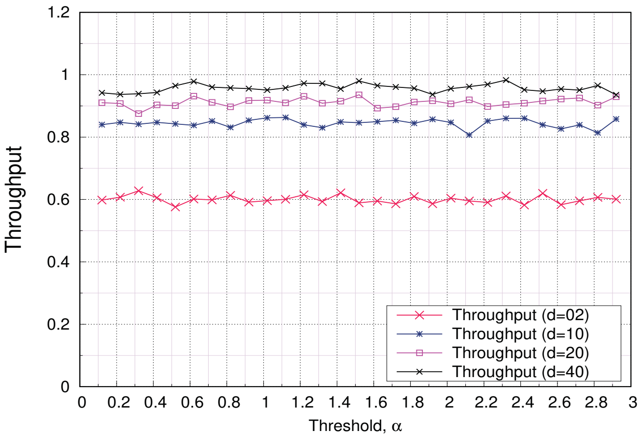

Fig. 4. Throughput Performance of Probabilistic Scheduler

From Figure 3, we observe that for a given threshold α, the average energy is the same for all d. Thus, it is clear that the average transmit energy is independent of delay deadline, whereas the throughput increases with the delay deadline d.

We use the same set of parameters (used for threshold based scheduler) for probabilistic scheduler to study the throughput and average energy performance which are plotted in Figure 4 and Figure 5. We note that the probabilistic detector does not depend on any threshold, and we draw the plots only to compare the performance against the threshold based scheduler (see Figures 2 and 3).

As the delay constraint d increases from 2 to 40, throughput

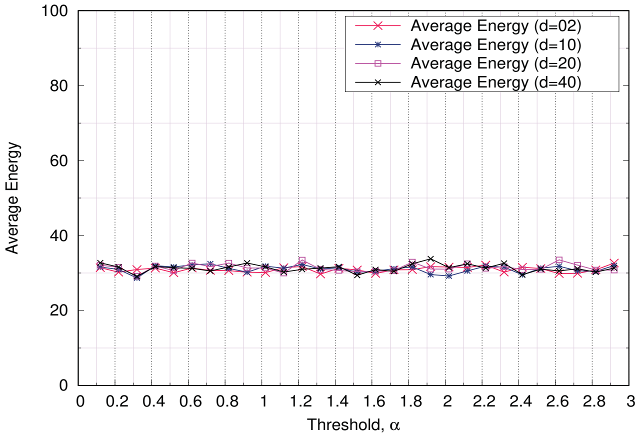

Fig. 5. Average Energy Performance of Probabilistic Scheduler

TABLE I
THROUGHPUT AND AVERAGE ENERGY PERFORMANCE OF PROBABILISTIC SCHEDULER

d	Throughput	Average Energy
2	0.61	30.52
10	0.83	31.41
20	0.91	31.51
40	0.94	31.56

increases in the case of probabilistic detector also. The reason being, the chance of finding a better channel is more in the case of a larger d, which results in energy efficiency, and better throughput (see Figure 4). Also, from Figure 5, we observe that the average energy is the same for all d. Table I, summarises the throughput and average energy performance of the probabilistic detector.

It is interesting to compare the threshold based scheduler and the probabilistic scheduler. To achieve a throughput of 0.6, threshold based scheduler requires an average energy of 50 Joules, whereas the probabilistic scheduler requires an average energy of about 30 Joules.

VI. CONCLUSIONS

In this paper, we have investigated the problem of transmitting packets such that each packet is required to reach the destination within a delay of d time–slots from its arrival. We have formulated the problem as a Markov Decision Process, and described the optimum policy using dynamic programming method. However, the complexity of this approach is huge, and hence, we propose two sub–optimal solutions, one based on a threshold on channel gain, and the other based on a probabilistic function of channel gain. We have evaluated both the algorithms for throughput and average transmission energy.

An analytical solution for the optimum policy is an open problem which can be taken as a future work.

REFERENCES

[1] B. E. Collins and R. L. Cruz, "Transmission policies for time varying channels with average delay constraints," in *1999 Allerton Conf. on Commun., Control., and Comp.*, 1999, pp. 709–717.

[2] R. A. Berry and R. G. Gallager, "Communication over fading channels with delay constraints," *IEEE Trans. Inf. Theory*, vol. 48, no. 5, pp. 1135–1149, May 2002.

[3] E. Uysal-Biyikoglu, B. Prabhakar, and A. E. Gamal, "Energy-efficient packet transmission over a wireless link," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 487–499, Aug. 2002.

[4] A. Fu, E. Modiano, and J. N. Tsitsiklis, "Optimal transmission scheduling over a fading channel with energy and deadline constraints," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 630–641, Mar. 2006.

[5] W. Chen and U. Mitra, "Energy efficient scheduling with individual packet delay constraints," in *Proceedings IEEE INFOCOM 2006. 25TH IEEE International Conference on Computer Communications*, Apr. 2006, pp. 1–12.

[6] W. Chen, M. J. Neely, and U. Mitra, "Energy efficient scheduling with individual packet delay constraints: Offline and online results," in *IEEE INFOCOM 2007 - 26th IEEE International Conference on Computer Communications*, May 2007, pp. 1136–1144.

[7] W. Chen, U. Mitra, and M. J. Neely, "Energy-efficient scheduling with individual packet delay constraints over a fading channel," *Wireless Networks*, vol. 15, no. 5, pp. 601–618, Jul. 2009.

[8] M. Zafer and E. Modiano, "Delay-constrained energy efficient data transmission over a wireless fading channel," in *2007 Information Theory and Applications Workshop*, Jan. 2007, pp. 289–298.

[9] X. Zhong and C. Z. Xu, "Online energy efficient packet scheduling with delay constraints in wireless networks," in *Proceedings of IEEE INFOCOM 2008*, April 2008.

[10] J. Lee and N. Jindal, "Energy-efficient scheduling of delay constrained traffic over fading channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1866–1875, Apr. 2009.

[11] R. Berry, E. Modiano, and M. Zafer, *Energy-efficient scheduling under delay constraints for wireless networks*. Morgan and Claypool Publishers, 2012, vol. 5, no. 2.