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# Optimal Downlink Scheduling and Power Allocation with Reconfiguration Delay

Gowri Muraleedharan, Vineeth B. S., Premkumar K.

***Abstract*—We consider a downlink scheduling problem in which the base station needs to serve a set of users. We consider a time-slotted system in which in each time-slot, the scheduler assigns service to a user. Whenever the scheduler changes the service from one user to the other, the base station has to switch it's configuration from that of the previous user to that of the current user. This incurs a delay, called "Reconfiguration delay." In this paper, we study the problem of scheduling in downlink with reconfiguration delay. Our objective is to design scheduling policies that optimally trade-off the average delay of data bits with the average transmission power. We consider an indpendent and identically distributed (IID) fading channel in which the channel gain in each time-slot varies according to a distribution. Also, the channel gain for each downlink user is independent of each other and follows the same distribution. We obtain an lower bound to the average power of transmission for any arrival rate vector in the stability region. For the case of constant channel state, we show that the outer bound is achievable using a variable frame drift + penalty policy. We also study the trade-off problem for the case of multiple channel states via simulations for two different scheduling policies.**

***Index Terms*—Wireless downlinks, Reconfiguration delay, Stability region, Queue stability, Power delay trade-off**

## I. Introduction

In this paper, we consider a simplified wireless downlink model where a base station serves different users by switching its service amongst the users. We note that such switching might incur a reconfiguration delay (see Figure 1 for an illustration). The reconfiguration delay is defined as the duration between the time at which the base station scheduler decides to serve a user and the time at which the actual service starts. Our motivation to study such systems stem from satellite systems with mechanically steered antenna, electronic beamforming, optical routers [1], and radio transceivers [2] which have reconfiguration delay. Modern mobile communication systems also have reconfiguration delay. We study the problem of scheduling of users for such systems. Wireless communication systems also have random connectivity between the users and the server which varies over time. It is important to control the switching between users and the service rate of transmissions so that random connectivity does not lead to a degradation in performance. Furthermore, service rate control is essential for minimizing the transmission power of a system. Motivated by these objectives, in this paper we consider the design of scheduling policies for wireless systems with reconfiguration delay with the objective of reducing the average delay of data as well as transmission power.

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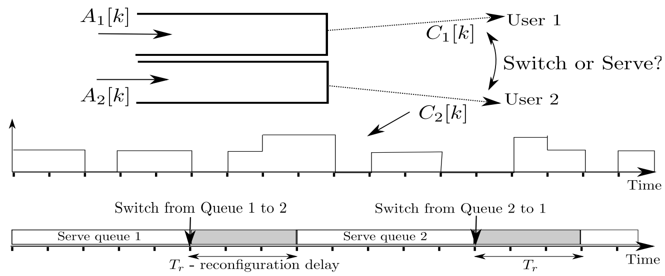


Fig. 1. An example wireless downlink with two users and a base station (BS). The data for the users have random bit arrivals $A_i[k]$ into their buffers at the BS. The BS can transmit at some rate to one user from its queue in a slot. The connections between the BS and the users are random and time varying. When the BS switches from serving one user to another there is a reconfiguration delay of $T_r$ slots.

### A. Prior work

It was observed by Celik et al. [3] that max-weight policies [4] may not be suitable for systems with reconfiguration delay as such policies may cause frequent switching between queues and hence may have reduced performance. For improved performance, Celik et al. [3] had proposed the variable frame max-weight (VFMW) policy. For the VFMW policy, switching between queues happens only at the ends of frames which are chosen as a suitable function of the queue length at the start of the frame. Hsieh et al. [5] proposed Queue-Biased Max Weight (QBMW) policy which has an intentional bias towards the currently served user thereby reducing the switching delay. QBMW policies were shown to improve the average delay performance of VFMW policies. In our prior work [6] we proposed a 1-lookahead policy, which was motivated by an approximate solution to a Markov decision process formulation of the scheduling problem. We also showed that the QBMW policy can be motivated from this approximate solution. We note these prior work only considered memoryless (IID) channels with multislot reconfiguration delay. Celik et al. [2] had proposed frame based dynamic control (FBDC) and m-lookahead myopic policies for correlated channels with a single slot reconfiguration delay. In our prior work [7] we had proposed scheduling policies for correlated channels and multislot reconfiguration delay. Other work such as [8], [9] also consider scheduling in networks with correlated random connectivity but without reconfiguration delay. We note that in all the above papers the aspect of rate control of the queue (and corresponding transmit power expenditure) of the system is not


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considered, which is what we address in this paper. A related work in this respect is that of Subhashini et al [10], which studied the problem of designing base station activation and rate allocation policies using time-scale separation. However, [10] considers reconfiguration cost rather than an explicit delay in reconfiguration.

### B. Outline and contributions

Our system model and problem statement are described in Section II. We present a queueing model with parallel queues and a single server to model the wireless downlink. The problem that we consider in this paper is a trade-off between average power and average delay; which is formulated as a constrained optimization problem. The feasibility of this problem is considered in Section III, where it is observed to be related to the notion of stability of the queueing system. Here we characterize the stability region of our queueing system and also obtain a lower bound to the average transmission power which is expended in stable operation of our system. In Section IV we propose a variable frame drift + penalty policy, which is our primary contribution. The policy is a combination of ideas from the VFMW policy of [3] as well as drift + penalty from [4]. A secondary contribution is a proof that the VFDP policy is optimal in the sense of achieving the lower bound obtained in Section III for the case of a fixed or time-invariant channel. We also propose a heuristic queue biased max-weight + penalty policy in Section V and evaluate the performance of the policies in Section VI through simulations. We conclude the paper in Section VII. We summarize the notation used in the paper here: (a) $\mathbb{Z}_+$ is the set of non-negative integers, (b) vectors are denoted using boldface (e.g. $\boldsymbol{s}$), (c) random variables are denoted using capital letters (e.g. $X$), (d) $\mathbb{E}X$ denotes expectation of a random variable $X$, and (e) $\boldsymbol{x}^T$ denotes the transpose of the vector $\boldsymbol{x}$.

## II. System model and problem statement

We consider a discrete time queueing model for the wireless downlink with $N$ users and a base-station as shown in Figure 1. The time slots are indexed by $k \in \mathbb{Z}_+$. The number of bits that are destined for user $i$ is assumed to arrive to the $i^{th}$ user's queue according to a random process. The random number of bits that arrive to the $i^{th}$ queue in the $k^{th}$ slot is denoted as $A_i[k]$. We assume that each user has an infinite queue buffer and these bits are queued up in the buffer, where they wait for service. We assume that $(A_i[k], k \geq 0)$ is independent and identically distributed (IID). We also assume that $A_i$-s are independent across users and $A_i[k] \leq A_{max}$. The arrival rate of user $i$ is denoted by $\lambda_i$, which is $\mathbb{E}A_i[0]$. The vector of all arrival rates is denoted as $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_N)^T$.

We assume that the $i^{th}$ user is connected to the base-station through a wireless channel, so that the $i^{th}$ user's queue can be modelled as being connected to the server through this channel.

TABLE I
GENERAL NOTATIONS

| Symbol | Description |
|---|---|
| N | Number of users |
| $A_i[k]$ | Number of bits per slot arriving to the $i^{th}$ user queue at time slot k |
| $A_{max}$ | Maximum possible bits that can arrive to a queue in a time slot |
| $\lambda_i$ | Arrival rate of user i |
| $\lambda$ | Vector of arrival rates |
| $C_i[k]$ | Channel state of $i^{th}$ user at time slot k |
| $\gamma_{i,c}$ | Probability of channel state c for $i^{th}$ user |
| $\mathcal{R}_i$ | Set of service rates for $i^{th}$ user |
| $T_r$ | Number slots required for reconfiguration |
| $S_i[k]$ | Number bits per slot serviced from user i at $k^{th}$ time slot |
| $P_i[k]$ | Power expenditure of $i^{th}$ user at time slot k |
| B | Bandwidth of wireless channel |
| $\sigma^2$ | Noise variance of receiver |
| $Q_i[k]$ | Queue length of $i^{th}$ user at time slot k |
| M[k] | Queue which is in service at time slot k |
| R[k] | Number of slots for which the current queue is in service |
| $\Gamma_c$ | Probability of channel state vector c |
| $T_f$ | Duration of $f^{th}$ frame |
| $t_f$ | First time slot of $f^{th}$ frame |

The channel has time varying state which models wireless fading. The channel state for the $i^{th}$ user in the $k^{th}$ slot is denoted as $C_i[k]$; the set of possible channel states for the $i^{th}$ user is denoted as $\mathcal{C}_i$. We assume that the channel states are IID with respect to time and independent across users. We denote the probability of the channel state being $c$ for the $i^{th}$ user as $\gamma_{i,c}, \forall c \in \mathcal{C}_i$. We note that $\sum_{c \in \mathcal{C}_i} \gamma_{i,c} = 1$. We use $\boldsymbol{C}[k] = (C_1[k], \ldots, C_N[k])^T$ to denote the channel state vector in slot $k$.

The server needs to switch service between the different queues in order to serve the bits from each queue. Furthermore, the server also needs to choose the rate of service for each queue. We assume that the set of service rates for the $i^{th}$ queue is $\mathcal{R}_i \subset \mathbb{Z}_+$; with a finite maximum value. In each slot, a scheduler makes decisions about whether the service needs to be switched from one queue to another, as well as the rate to be used for service. We note that switching service from one queue to another incurs a reconfiguration delay of $T_r$ slots; during this reconfiguration time there is no service from the queue that we have switched to. We denote the rate of service for the $i^{th}$ queue in slot $k$ as $S_i[k]$ (bits/slot). We note that $S_i[k] = 0$ if the current queue under service is not $i$ or if the current queue under service is $i$, but slot $k$ is within the reconfiguration duration after a switch has been made. We assume that when a rate of $S_i[k]$ is used for queue $i$, then a transmission power of $P_i[k]$ is incurred which is a function of the rate $S_i[k]$ and channel state $C_i[k]$. For example, $S_i[k], C_i[k]$, and $P_i[k]$ are related as $S_i[k] = B \log_2 \left(1 + \frac{C_i^2[k] P_i[k]}{\sigma^2}\right)$, where $B$ is the bandwidth of the wireless channel and $\sigma^2$ is the noise variance at the receiver (the transmitter and receiver has channel state information and noise is additive white Gaussian). In general, we say that $P_i[k] = P_i(S_i[k], C_i[k])$.

We denote the number of bits which are queued at the $i^{th}$



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buffer at the beginning of slot $k$ as $Q_i[k]$. Then, we have that

$$Q_i[k+1] = (Q_i[k] - S_i[k])^+ + A_i[k], \qquad (1)$$

where $(.)^+ = \max(.,0)$. We use $\boldsymbol{Q}[k] = (Q_1[k], Q_2[k], \dots, Q_N[k])^T$ to denote the vector of queue lengths at $k$ and $\boldsymbol{S}[k]$ for the vector of service rates. We note that only one component of $\boldsymbol{S}[k]$ would be non-zero at any time $k$ or $\boldsymbol{S}[k] = \boldsymbol{0}$.

We also introduce some additional notation: (a) $M[k] \in \{1, 2, \dots, N\}$ denotes the queue which is in service in slot $k$ and (b) $R[k]$ denotes the number of slots for which the current queue is being served. We note that unless $R[k] > T_r$ no service happens from the current queue. We also note that if $M[k] \neq M[k-1]$ then $R[k] = 1$, otherwise $R[k] = R[k-1] + 1$.

A scheduling policy $\mu$ is a sequence $((M[k], \boldsymbol{S}[k]), k \geq 0)$ of decisions of which queue to serve and what service rate to use for that queue. We assume that at the beginning of every slot the channel state vector $\boldsymbol{C}[k]$ is known to the scheduler for making a decision about $M[k]$ and $\boldsymbol{S}[k]$.

For a scheduling policy $\mu$, we are interested in two measures of performance: the total average queue length and the total average power which are defined as follows. The total average queue length $\overline{q}(\mu)$ for a scheduling policy $\mu$ is:

$$\limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{k=0}^{K-1}\sum_{i=1}^{N} Q_i[k] \middle| \boldsymbol{Q}[0]\right].$$

In this paper, we consider the total average queue length in place of average delay since for a fixed total arrival rate ($\sum_i \lambda_i$) the average delay is proportional to the total average queue length by Little's law. The total average power $\overline{p}(\mu)$ for a scheduling policy $\mu$ is:

$$\limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\sum_{k=0}^{K-1}\sum_{i=1}^{N} P_i[k] \middle| \boldsymbol{Q}[0]\right].$$

We note that it is desirable to obtain a policy $\mu$ such that $\overline{q}_\mu$ and $\overline{p}_\mu$ are as small as possible. However, it is intuitive that we have a trade-off between $\overline{q}_\mu$ and $\overline{p}_\mu$ since a policy that uses a larger service rate would have a smaller value of $\overline{q}_\mu$ but larger $\overline{p}_\mu$. Therefore, the problem that we are interested in is the characterization of this trade-off between $\overline{q}_\mu$ and $\overline{p}_\mu$. In this paper, we consider this problem for stationary policies. Stationary policies are such that the $M[k]$ and $\boldsymbol{S}[k]$ are chosen as a function only of the current state. The current state of the system at the beginning of slot $k$ is defined to be the tuple $(M[k-1], \boldsymbol{Q}[k], \boldsymbol{C}[k])$.

The trade-off problem is then the following optimization problem:

$$\begin{aligned} \underset{\mu}{\text{minimize}} \quad & \overline{p}_\mu, \\ \text{such that} \quad & \overline{q}_\mu \leq q_c. \end{aligned} \qquad (2)$$

When the above problem is feasible, the optimal value if it exists is denoted as $p^*(q_c)$ and an optimal policy as $\mu^*$. In the next section, we consider the question of feasibility of this problem.

## III. Feasibility of tradeoff problem (2)

Suppose $\lambda_i > \max \mathcal{R}_i$, i.e., the arrival rate into the $i^{th}$ queue is more than the maximum service rate from the $i^{th}$ queue, then intuitively under any policy $\mu$ it is not possible to have a finite total average queue length for our system. It is then also intuitive that there exists a finite set $\boldsymbol{\Lambda}$ of arrival rate vectors $\boldsymbol{\lambda}$ for which (2) is feasible. Our interest is therefore to solve the (2) only for $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$. We consider

$$\begin{aligned} \underset{\mu}{\text{minimize}} \quad & \overline{p}_\mu, \\ \text{such that} \quad & \overline{q}_\mu < \infty. \end{aligned} \qquad (3)$$

We note that if (2) is feasible for some $q_c$, then the above problem is feasible.

Consider the following optimization problem:

$$\begin{aligned} \underset{\beta_{\boldsymbol{r},\boldsymbol{c}}}{\text{minimize}} \quad & \sum_{\boldsymbol{c}} \Gamma_{\boldsymbol{c}} \sum_{\boldsymbol{r}} \beta_{\boldsymbol{r},\boldsymbol{c}} P(\boldsymbol{r},\boldsymbol{c}), \\ \text{such that} \quad & \sum_{\boldsymbol{c}} \Gamma_{\boldsymbol{c}} \sum_{\boldsymbol{r}} \beta_{\boldsymbol{r},\boldsymbol{c}} \boldsymbol{r} \geq \boldsymbol{\lambda}, \\ & \beta_{\boldsymbol{r},\boldsymbol{c}} \geq 0, \forall \boldsymbol{r}, \boldsymbol{c}, \text{ and } \sum_{\boldsymbol{r}} \beta_{\boldsymbol{r},\boldsymbol{c}} = 1. \end{aligned} \qquad (4)$$

Here $\boldsymbol{c}$ is a channel state vector which is an element of the Cartesian product $\times_i \mathcal{C}_i$, $\boldsymbol{r}$ is a service rate vector which is an element of the Cartesian product $\times_i \mathcal{R}_i$, and for $\boldsymbol{c} = (c_1, c_2, \dots, c_N)^T$, $\Gamma_{\boldsymbol{c}} = \Pi_i \gamma_{i,c_i}$ is the probability of the channel state vector $\boldsymbol{c}$. The function $P(\boldsymbol{r},\boldsymbol{c}) = \sum_i P_i(r_i, c_i)$. We note that $\beta_{\boldsymbol{r},\boldsymbol{c}}$ can be interpreted as the fraction of time a rate vector $\boldsymbol{r}$ is used when the channel state vector is $\boldsymbol{c}$. The optimal value of the above problem is denoted as $p^*(\boldsymbol{\lambda})$.

For a given $\boldsymbol{\lambda}$ we note that (4) is a relaxed form of (3), since the constraint on the average queue length has been replaced by a necessary condition (average service rate vector dominates the arrival rate vector)[1]. We denote the set of all $\boldsymbol{\lambda}$ for which (3) is feasible as $\boldsymbol{\Lambda_1}$. Since (3) is a relaxed form of (2) we have the following result.

*Proposition 3.1:* The optimal value $p^*(\boldsymbol{\lambda})$ of (4) is a lower bound to the optimal value of (2). The set of arrival rates $\boldsymbol{\Lambda_1}$ is a superset (or outer bound) of $\boldsymbol{\Lambda}$.

We note that (4) is a linear programming problem in the decision variables $\beta_{\boldsymbol{r},\boldsymbol{c}}, \forall r$. This linear programming problem can be solved using standard numerical packages in order to obtain $p^*(\boldsymbol{\lambda})$ as well as to check if a given $\boldsymbol{\lambda} \in \boldsymbol{\Lambda_1}$. Consider an example with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0, 0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The channel state probabilities are assumed to be same for queues and are $\gamma_{1,0} = 0.2, \gamma_{1,0.5} = 0.3$, and $\gamma_{1,1} = 0.5$. In Figure 2 we illustrate a set of $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}_1$. We obtain this set of $\boldsymbol{\lambda}$ by exhaustively checking for the feasibility of the linear program (4) for a given $\boldsymbol{\lambda}$. In Figure 3 we illustrate

[1]There are some technical issues here which we summarize. In deriving this relaxed optimization problem, the dominance of a system with reconfiguration delay by a system without reconfiguration delay is also used. This is because any stationary policy for the system with reconfiguration delay is a non-stationary policy for the system without reconfiguration delay, where the policy does not serve for $T_r$ slots after a switch. Then, as in [11], it can be shown that (4) is a relaxed problem for the system without reconfiguration delay over all policies, and hence it is a lower bound for the system with reconfiguration delay.



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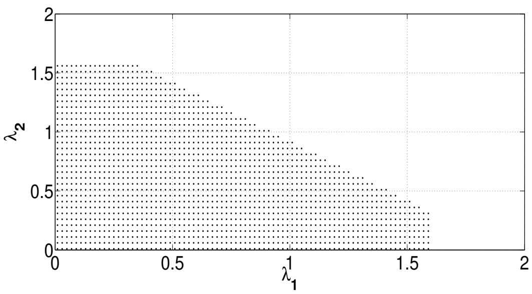


Fig. 2. Illustration of a subset of arrival rate vectors $\boldsymbol{\lambda}$ that belongs to $\boldsymbol{\Lambda}_1$ for a system with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0, 0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The channel state probabilities are assumed to be same for queues and are $\gamma_{1,0} = 0.2, \gamma_{1,0.5} = 0.3$, and $\gamma_{1,1} = 0.5$. The bandwidth $B$ and $\sigma^2$ are chosen to be 1000 and 1 respectively.

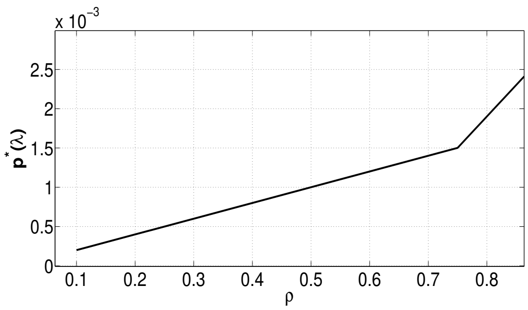


Fig. 3. Illustration of $p^*(\boldsymbol{\lambda})$ for $\boldsymbol{\lambda} = \rho \times (0, 1)^T, \rho \in (0, 1)$ for a system with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0, 0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The channel state probabilities are assumed to be same for queues and are $\gamma_{1,0} = 0.2, \gamma_{1,0.5} = 0.3$, and $\gamma_{1,1} = 0.5$. The bandwidth $B$ and $\sigma^2$ are chosen to be 1000 and 1 respectively.

$p^*(\boldsymbol{\lambda})$ for $\boldsymbol{\lambda} = \rho \times (1, 1)^T$ where $\rho \in (0, 1)$ for the same example system used above.

To summarize, the trade-off problem (2) should be considered for $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$. However, we have an outer bound $\boldsymbol{\Lambda}_1$ for $\boldsymbol{\Lambda}$. In this paper, we study this trade-off problem using simulations in Section VI.

However, for the case where $|\mathcal{C}_i| = 1$ for every user $i$ (that is the channel state is fixed or we have a time-invariant channel) we show that all $\boldsymbol{\lambda} \in \boldsymbol{\Lambda_1}$ (except possibly those at the *boundary* of $\boldsymbol{\Lambda}_1$) can be achieved. However, this has not been extended to the case of $|\mathcal{C}_i| > 1$ and is part of future work. For the case of $|\mathcal{C}_i| = 1, \forall i$, we also show that $p^*(\boldsymbol{\lambda})$ can be achieved arbitrarily closely for $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}_1$ and in the process obtain a characterization of $p^*(q_c)$ for (2).

## IV. Variable frame drift + penalty policy

In this section, we propose a variable frame drift + penalty (VFDP) scheduling policy for our system. VFDP policy is an extension of the VFMW policy of Celik et al. [2] to the case with rate control. VFDP is a parameterized family of policies which trade-off the average queue length with average power. For the case where the channel is constant, we also show that the VFDP policy achieves average power values arbitrarily close to $p^*(\boldsymbol{\lambda})$ for $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}_1$.

The VFDP policy is implemented over frames; in each frame one queue is served. The intuition is that by making the frame durations large, the switching between queues can be reduced. Let us denote that slots at which frames start as $t_0, t_1, \ldots, t_f, \ldots, f \in \mathbb{Z}_+$, with $t_0 = 0$. The duration of the $f^{th}$ frame is denoted as $T_f$, e.g., $T_0 = t_1 - t_0$. In the following we will choose $T_f$ as a slowly growing function of the total queue length at the frame-start time $t_f$.

The VFDP policy is defined as follows. The decision to switch between queues is only taken at the frame start slots. However, the rate of service from the queue that is chosen for a particular frame has to be decided in every slot in the frame duration. We assume that the first queue was in service at the start of system operation so that $M[-1] = 1$. At the start of the $f^{th}$ frame we compute the following metric $E_i$ for each queue $i$. If $i = M[t_f - 1]$ then

$$E_i = \min_{s_c[\tau], \forall \tau} \Bigg[ -2Q_i[t_f] \left\{ \sum_{\tau=t_f}^{t_{f+1}-1} \left\{ \sum_{c \in \mathcal{C}_i} \gamma_{i,c} s_c[\tau] - \lambda \right\} \right\} + V \sum_{\tau=t_f}^{t_{f+1}-1} \sum_{c \in \mathcal{C}_i} \gamma_{i,c} P_i(s_c[\tau], c) \Bigg],$$

where the optimization is carried out over the variables $s_c[\tau]$ for every $\tau \in \{t_f, \ldots, t_{f+1} - 1\}$. The intuition behind this metric comes from a drift + penalty expression obtained by considering Lyapunov drift embedded at the frame start epochs. Also if $i \neq M[t_f - 1]$ then

$$E_i = \min_{s_c[\tau]} \Bigg[ -2Q_i[t_f] \left\{ \sum_{\tau=t_f+T_r}^{t_{f+1}-1} \left\{ \sum_{c \in \mathcal{C}_i} \gamma_{i,c} s_c[\tau] - \lambda \right\} \right\} + V \sum_{\tau=t_f+T_r}^{t_{f+1}-1} \sum_{c \in \mathcal{C}_i} \gamma_{i,c} P_i(s_c[\tau], c) \Bigg],$$

where we consider $s_c[\tau]$ for every $\tau \in \{t_f + T_r, \ldots, t_{f+1} - 1\}$ since there is a reconfiguration delay. The VFDP policy is defined in Algorithm 1. We note

**Algorithm 1** VFDP policy

**if** slot index $k ==$ frame start time $t_f$ **then**
- a: Calculate the metrics $E_i$.
- b: Switch to a queue $i^*$ where

$$i^* = \arg\min_i E_i.$$

- c: $M[k] = i^*$
- d: $t_{f+1} = t_f + T_r + \left(\sum_i Q_i[t_f]\right)^\alpha, \alpha \in (0, 1)$.

**for** every slot $k$ **do** obtain the service rate $S[k]$ as follows. Suppose $M[k] = i$ and the channel state for the $i^{th}$ queue is $C_i[k]$,

$$S[k] = \arg\min_s \left[-2Q_i[t_f] \{s - \lambda\} + VP_i(s, C_i[k])\right],$$

where $t_f$ is the frame start time such that $k \in \{t_f, \ldots, t_{f+1} - 1\}$.



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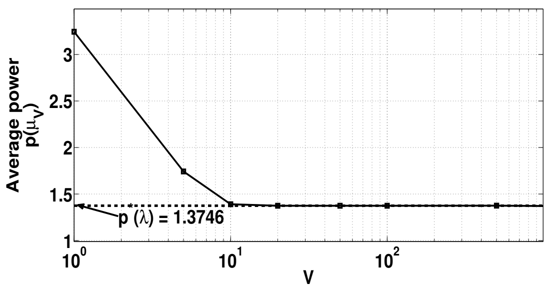


Fig. 4. The plot of $p(\mu_V)$ versus $V$ for a sequence of VFDP policies (with $\alpha = 0.5$) for a system with $|\mathcal{C}_i| = 1, \forall i$. The system has $N = 2$, $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}, T_r = 3, B = 1$, and $\sigma^2 = 1$. The arrival rate vector is $\boldsymbol{\lambda} = (0.4, 0.4)^T$.

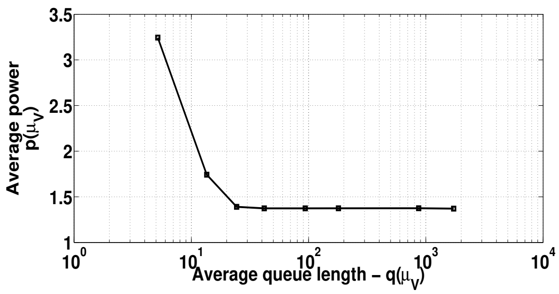


Fig. 5. The plot of $p(\mu_V)$ versus $q(\mu_V)$ for a sequence of VFDP policies (with $\alpha = 0.5$) for a system with $|\mathcal{C}_i| = 1, \forall i$. The system has $N = 2$, $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}, T_r = 3, B = 1$, and $\sigma^2 = 1$. The arrival rate vector is $\boldsymbol{\lambda} = (0.4, 0.4)^T$ for which $p^*(\boldsymbol{\lambda}) = 1.3746$. We note that this is an upper bound to $p^*(q_c)$.

that the VFDP policy is parameterized by: (a) $\alpha \in (0, 1)$ which decides the frame duration and (b) $V$ which trades-off the importance of minimizing power and increasing the service rate. We have the following result for the VFDP policy which motivates its use as a scheduling policy for our system.

*Proposition 4.1:* Consider the case where $|\mathcal{C}_i| = 1$ for every $i$ for a system with arrival rate vector $\boldsymbol{\lambda}$. Let us define a sequence of VFDP policies $\mu_V$ for a sequence $V \to \infty$. Then, we have that $p(\mu_V) \to p^*(\boldsymbol{\lambda})$, with $q(\mu_V) < \infty$.

The proof of this proposition is given in Appendix A. It also then follows that any point in the interior of $\boldsymbol{\Lambda}_1$ is stable for the case of a fixed channel state. We illustrate the above property of VFDP policies using a simulation in Figure 4. We observe that as $V \to \infty$ the average powers achieved by the VFDP policies approach the lower bound $p^*(\boldsymbol{\lambda})$. The error bar length of this plot is 0.0034 which is very small. We also note that since VFDP is some policy we can also obtain an upper bound on $p^*(q_c)$ which is shown in Figure 4. We also observe that as $p^*(\boldsymbol{\lambda})$ is approached, $q(\mu_V) < \infty$ and $q(\mu_V)$ is also proportional to $V$ which is the usual behaviour observed for drift + penalty algorithms [4].

## V. Queue biased max-weight + penalty policy

In this section, motivated by the approach in [5] and our prior work [6], we propose a queue biased max-weight + penalty (QBMWP) policy. We note that (2) can be written as

$$\underset{\mu}{\text{minimize}}\, \overline{p}_\mu + L\overline{q}_\mu, \text{ or,}$$
$$\underset{\mu}{\text{minimize}}\, \overline{q}_\mu + V\overline{p}_\mu,$$

where $L$ is a Lagrange multiplier and $V$ is $\frac{1}{L}$, both strictly greater than 0. Then in every slot $k$, we define the following metrics. Suppose $M[k-1] = i$, then

$$W_i[k] = \min_{s_c[n]} \sum_{n=0}^{T_r} \left[ \sum_{c \in \mathcal{C}_i} \gamma_{i,c} \left( -2Q_i[k]s_c[n] + VP_i(s_c[n], c) \right) \right].$$

For a queue $j \neq i$,

$$W_j[k] = \min_{s_c} \left[ \sum_{c \in \mathcal{C}_i} \gamma_{i,c} \left( -2Q_i[k]s_c + VP_i(s_c, c) \right) \right].$$

We now propose our heuristic QBMWP based on the above metrics in Algorithm 2.

---

**Algorithm 2** QBMWP policy

---

**for** every slot $k$ **do**

1: Calculate the weights $W_j[k]$, $j \in \{1, 2, \ldots, N\}$.

2: If $W_j[k] > W_i[k]$, then switch to the $j^{th}$ queue (i.e., $M[k] = j$), else stay with the $i^{th}$ queue.

3: For the currently served queue (let this be $i$), the service rate $S[k]$ is chosen as

$$S[k] = \arg\min_s \left[ -2Q_i[k] \{s - \lambda\} + VP_i(s, C_i[k]) \right],$$

if $R[k] > T_r$, otherwise $S[k] = 0$.

---

Currently, we do not have a proof of whether a sequence of QBMWP policies achieve $p^*(\boldsymbol{\lambda})$ even for $|\mathcal{C}_i = 1|, \forall i$. In the next section, we study the trade-off performance of both policies for systems with multiple channel states using simulations.

## VI. Simulation Results

In this section we illustrate and compare the trade-off performance of VFDP and QBMWP policies for the case of multiple channel states. In Figure 6 we consider a system with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0, 0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The channel state probabilities are assumed to be same for queues and are $\gamma_{1,0} = 0.2, \gamma_{1,0.5} = 0.3$, and $\gamma_{1,1} = 0.5$. The bandwidth $B$ and $\sigma^2$ are chosen to be 1 and 1 respectively. The arrival rate vector is $(0.4, 0.4)^T$. The different points on the trade-off curve are obtained by varying the parameter $V$. We observe that as $V$ increases beyond 50000, the simulations require iterations of the order of $10^9$ to converge. Such values of $V$ are excluded from our results. Even though the QBMWP policy does not explicitly make use of frames, we observe that the trade-off performance is worse compared to that of VFDP



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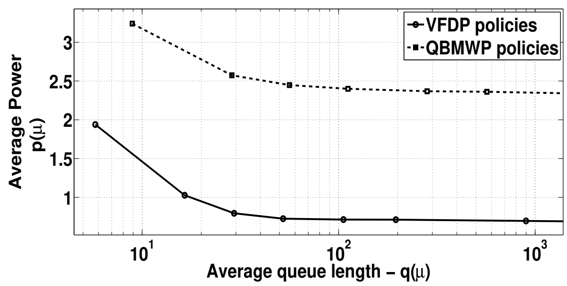


Fig. 6. The plot of $p(\mu_V)$ versus $q(\mu_V)$ for a sequence of VFDP ($\alpha = 0.5$) and QBMWP policies for a system with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0, 0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The $\gamma_{i,c}$ are assumed to be same for all queues and are $\gamma_{1,0} = 0.2, \gamma_{1,0.5} = 0.3$, and $\gamma_{1,1} = 0.5$. The bandwidth $B$ and $\sigma^2$ are chosen to be 1 and 1 respectively. The arrival rate vector is $(0.4, 0.4)^T$.

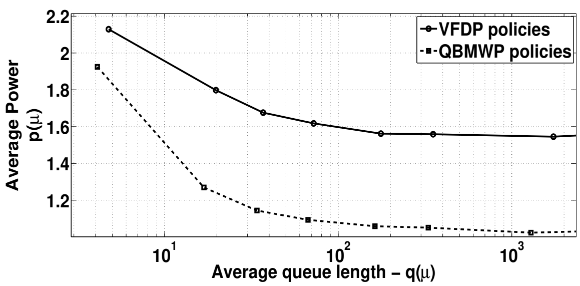


Fig. 7. The plot of $p(\mu_V)$ versus $q(\mu_V)$ for a sequence of VFDP ($\alpha = 0.9$) and QBMWP policies for a system with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$. The $\gamma_{i,c}$ are assumed to be same for all queues and are $\gamma_{1,0.5} = 0.4$, and $\gamma_{1,1} = 0.6$. The bandwidth $B$ and $\sigma^2$ are chosen to be 1 and 1 respectively. The arrival rate vector is $(0.1, 0.5)^T$.

in this case. We also consider another example in Figure 7 with $N = 2$, $\mathcal{C}_1 = \mathcal{C}_2 = \{0.5, 1\}$, and $\mathcal{R}_1 = \mathcal{R}_2 = \{0, 1, 2\}$ (other parameter choices are given in the figure). In this case we observe that QBMWP has a better trade-off performance than VFDP, showing that it is necessary to thoroughly characterize the performance of these policies analytically in order to decide which policy should be used in practice.

## VII. Conclusions and Future work

In this paper, we consider the problem of designing scheduling policies for trading off average power with average delay (or average queue length) for a queueing system with reconfiguration delay. We obtained an outer bound to the set of arrival rates for which the average queue length is finite; this is obtained using a linear programming problem. The linear program also provides a lower bound to the minimum average power for a given arrival rate vector. We also proposed two policies, VFDP and QBMWP which trade-off the average power with average queue length. We showed that the VFDP policies achieves the minimum average power for the case of a system with a fixed channel state for all users and compared the performance of these policies using simulations. In future work, we plan to investigate whether the VFDP policies or the QBMWP policies have the property of achieving the minimum average power for a given arrival rate vector for systems with multiple channel states. Our conjecture is that the lower bound that we obtain for the minimum average power as well as the region $\mathbf{\Lambda}_1$ are both loose and needs further investigation. We also plan to analyze the performance of our policies for systems with $N > 2$.

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## Appendix A
## Proof of proposition 4.1

The proof uses a Lyapunov drift + penalty technique motivated by the proof of stability of VFMW in [2]. This is extended with the usual average power bounding technique used in [4]. Because of space constraints, we only provide an outline of the proof here. We consider the queue lengths at the frame start slots. We have that

$$Q_i[t_{f+1}] \leq \left( Q_i[t_f] - \sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau] \right)^+ + \sum_{\tau=t_f}^{t_{f+1}-1} A_i[\tau].$$

We use a Lyapunov function $L(\boldsymbol{Q}) = \sum_i Q_i^2$. Therefore

$$\begin{aligned}(Q_i[t_{f+1}])^2 \quad \leq \quad & 2 \left( Q_i[t_f] - \sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau] \right)^+ \sum_{\tau=t_f}^{t_{f+1}-1} A_i[\tau] \\ & + \left( Q_i[t_f] - \sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau] \right)^2 \\ & + \left( \sum_{\tau=t_f}^{t_{f+1}-1} A_i[\tau] \right)^2 .\end{aligned}$$



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Using $A_i[\tau] \leq A_{max}$ and $S_i[\tau] \leq \max \mathcal{R}_i$, we can then show that

$$(Q_i[t_{f+1}])^2 \leq -2Q_i[t_f]\left[\sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau] - \sum_{\tau=t_f}^{t_{f+1}-1} A_i[\tau]\right] + BT_f^2,$$

where $B$ is a non-negative constant. The expected Lyapunov drift is then

$$\mathbb{E}\left[L(\boldsymbol{Q}[t_{f+1}]) - L(\boldsymbol{Q}[t_f]) | \boldsymbol{Q}[t_f], M[t_f-1]\right],$$

which can be bounded above as

$$-2\sum_i Q_i[t_f]\mathbb{E}_{t_f}\left[\sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau] - \sum_{\tau=t_f}^{t_{f+1}-1} A_i[\tau]\right] + NBT_f^2,$$

where $\mathbb{E}_{t_f}$ denotes the conditional expectation with respect to the state $\boldsymbol{Q}[t_f], M[t_f-1]$. Using the usual idea of drift + penalty we consider

$$-2\sum_i Q_i[t_f]\left[\mathbb{E}_{t_f}\left[\sum_{\tau=t_f}^{t_{f+1}-1} S_i[\tau]\right] - T_f\lambda_i\right] + NBT_f^2 + V\sum_i \sum_{\tau=t_f}^{t_{f+1}-1} \mathbb{E}_{t_f} P_i(S_i[\tau]).$$

Here the $P_i(.)$ is a function only of the rate $S_i[\tau]$ since the channel state is fixed. We note that this is the metric used for the VFDP policy in Section IV for the case of a single channel state. For brevity, we will use vector notation. Let $\boldsymbol{S}[\tau] = (S_i[\tau])^T$ and $P(\boldsymbol{S}[\tau]) = \sum_i P_i(S_i[\tau])$. Then the above drift + penalty expression can be written as

$$-2\boldsymbol{Q}[t_f]^T\left[\sum_{\tau=t_f}^{t_{f+1}-1} \mathbb{E}_{t_f}\boldsymbol{S}[\tau] - T_f\boldsymbol{\lambda}\right] + NBT_f^2 + V\sum_i \sum_{\tau=t_f}^{t_{f+1}-1} \mathbb{E}_{t_f} P(\boldsymbol{S}[\tau]).$$

We note that the rate vectors $\boldsymbol{S}[\tau]$ have a particular structure; within each frame $\boldsymbol{S}[\tau]$ is a vector which is either all zero, or which is non-zero in just one component $i$. We note that at every $t_f$ the VFDP policy chooses that $i$ which achieves the minimum value of the above expression. Also within each frame the VFDP policy chooses that rate (or rate vector) which achieves the minimum value of the term within the sum over $\tau$. Therefore, the drift + penalty expression above can be further bounded above using the drift + penalty for a randomized policy which we define as follows.

Let $\mathcal{R}$ be the set of all rate vectors and let $\mathcal{R}_i$ denote the rate vectors which are non-zero in the $i^{th}$ component. Suppose we solve (4) and obtain $\beta_{\boldsymbol{r}}$ (here again the dependence on channel state is suppressed). Let $\beta_i = \sum_{\boldsymbol{r}\in\mathcal{R}_i} \beta_{\boldsymbol{r}}$. Then the above drift + penalty expression can be bounded above as

$$NBT_f^2 - 2\boldsymbol{Q}[t_f]^T\left[\sum_i \beta_i \sum_{\tau=t_f+T_r}^{t_{f+1}-1} \sum_{\boldsymbol{r}\in\mathcal{R}_i} \frac{\beta_{\boldsymbol{r}}}{\beta_i}\boldsymbol{r} - \boldsymbol{\lambda}T_f\right] + V\sum_i \beta_i \sum_{\tau=t_f}^{t_{f+1}-1} \sum_{\boldsymbol{r}\in\mathcal{R}_i} \frac{\beta_{\boldsymbol{r}}}{\beta_i} P(\boldsymbol{r}).$$

Here the bound holds because the rate terms are positive and so the terms in the sum for $\tau \in \{t_f, \ldots, t_f + T_r - 1\}$ can be left out and the power expended would be zero if there is a switch. Thus, for our policy we have that

$$\mathbb{E}_{t_f}\left[L(\boldsymbol{Q}[t_{f+1}]) - L(\boldsymbol{Q}[t_f])\right] + V\sum_{\tau=t_f}^{t_{f+1}-1}\sum_i P_i[\tau] \leq NBT_f^2 - 2\boldsymbol{Q}[t_f]^T\left[\sum_{\tau=t_f+T_r}^{t_{f+1}-1}\sum_{\boldsymbol{r}\in\mathcal{R}} \beta_{\boldsymbol{r}}\boldsymbol{r} - \boldsymbol{\lambda}T_f\right] + V\sum_{\tau=t_f}^{t_{f+1}-1}\sum_{\boldsymbol{r}\in\mathcal{R}} \beta_{\boldsymbol{r}} P(\boldsymbol{r}),$$

since $\sum_i \beta_i \sum_{\boldsymbol{r}\in\mathcal{R}_i} \frac{\beta_{\boldsymbol{r}}}{\beta_i}(.) = \sum_{\boldsymbol{r}} \beta_{\boldsymbol{r}}(.)$.

We note that if $\boldsymbol{\lambda}$ is in the interior of $\boldsymbol{\Lambda}_1$ then it is possible to obtain $\beta_{\boldsymbol{r}}$ for a $\boldsymbol{\lambda} + \boldsymbol{\epsilon}$ where $\boldsymbol{\epsilon}$ is a vector of all $\epsilon > 0$. For such a $\beta_{\boldsymbol{r}}$ we have that the expected drift + penalty is

$$\leq NBT_f^2 - 2\boldsymbol{Q}[t_f]^T\left[(T_f - T_r)(\boldsymbol{\lambda}+\boldsymbol{\epsilon}) - \boldsymbol{\lambda}T_f\right] + VT_f p^*(\boldsymbol{\lambda}+\boldsymbol{\epsilon}).$$

Using the technique of telescoping sums [4] across the frame indices $f \in \{0, \ldots, K-1\}$ we have that

$$V\sum_{\tau=0}^{t_K-1}\sum_i P_i[\tau] \leq L(\boldsymbol{Q}[0]) + \sum_f NBT_f^2 + Vt_K p^*(\boldsymbol{\lambda}+\boldsymbol{\epsilon}) - 2\sum_f \boldsymbol{Q}[t_f]^T\left[(T_f - T_r)(\boldsymbol{\lambda}+\boldsymbol{\epsilon}) - T_f\boldsymbol{\lambda}\right]$$

As in [2], if $T_f$ is chosen as $T_r + (\sum_i Q_i[t_f])^\alpha$, then it is possible to prove that there exists a non-negative constant $C_0$ such that the above expression can be written as

$$V\sum_{\tau=0}^{t_K-1}\sum_i P_i[\tau] \leq L(\boldsymbol{Q}[0]) + C_0\sum_f T_f + Vt_K p^*(\boldsymbol{\lambda}+\boldsymbol{\epsilon}).$$

Therefore the average power of the VFDP policy can be obtained as

$$p(\mu_V) \leq \lim_{K\to\infty} \frac{L(\boldsymbol{Q}[0]) + C_0\sum_f T_f}{Vt_K} + p^*(\boldsymbol{\lambda}+\boldsymbol{\epsilon}) = \frac{C_0}{V} + p^*(\boldsymbol{\lambda}+\boldsymbol{\epsilon}).$$

By choosing $\epsilon \downarrow 0$ as $V \to \infty$ it is then possible to show that $\lim_{V\to\infty} p(\mu_V) = p^*(\boldsymbol{\lambda})$. The proof of stability, i.e., $q(\mu_V) < \infty$ is similar to that in [2] and is not presented here.



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